RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2019

SECOND YEAR [BATCH 2018-21]

MATHEMATICS FOR INDUSTRIAL CHEMISTRY [General]

Paper : III

Date : 21/12/2019

Time : 11 am – 2 pm

Full Marks : 75

[Use a separate Answer Book for each group]

Group – A

Answer any five questions from Question nos. 1 to 8 : [5×5]

1. If $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents two intersecting lines , then find the angle between them. (5)

2. Find the equation of the plane bisecting the angle between the planes $3x-6y+2z+5=0$ and $4x-12y+3z-3=0$ which contains the origin. (5)

3. Find the equation of the plane through the straight line $x+y-2z+4=0=3x-y+2z+1$ and parallel to the straight line $\frac{x-2}{2}=\frac{y-2}{2}=\frac{z-1}{-1}$. (5)

4. Find the shortest distance between the straight lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$. (5)

5. Show that $\frac{a}{y-z}+\frac{b}{z-x}+\frac{c}{x-y}=0$ represents a pair of intersecting planes whose line of intersection is equally inclined to the axes. (5)

6. On the conic $r=\frac{21}{5-2\cos\theta}$ find the point with the least radius vector. (5)

7. Show that the polar of any point on the circle $x^2+y^2-2ax-3a^2=0$ with respect to the circle $x^2+y^2+2ax-3a^2=0$ will be a tangent to the parabola $y^2=-4ax$. (5)

8. Transform the following equation to its canonical form.
$25x^2-14xy+25y^2+64x-64y-224=0$. (5)

Answer any five questions from Question nos. 9 to 16 : [5×5]

9. Show that the following differential equation is exact and using this property find the general solution.
$2(y+1)e^x\,dx+2\left(e^x-2y\right)dy=0$. (5)

10. Solve : $\frac{dy}{dx}+\frac{1}{3}y=xy^3$. (5)

11. Find the orthogonal trajectories of the family of parabolas $y^2=4ax$, a being the parameter. (5)

12. Apply the method of variation of parameters to solve
$\frac{d^2y}{dx^2}+4y=\tan 2x$. (5)

13. Solve : $y=px+\sqrt{a^2p^2+b^2}$. (5)

14. Show that the differential equation $\frac{dy}{dx}+P(x)y=Q(x)y^n$, where $n\neq 1$ can be reduced to a linear differential equation using the substitution $z=y^{n-1}$. (5)

15. Solve : $x\,dx+y\,dy+\frac{x\,dy-y\,dx}{x^2+y^2}=0$, given that y=1 , when x=1. (5)

16. Solve : $\frac{dy}{dx}+\frac{3x+2y-5}{2x+3y-5}=0$. (5)

Group – B

***Answer any five questions from Question Nos. 17 to 24* :** [5×5]

17. (a) Let $S=\{(x,y,z)\in\mathbb{R}^3: y=0, z=0\}$ and $T=\{(x,y,z)\in\mathbb{R}^3: x=0, y=0\}$ be two subspaces of $\mathbb{R}^3$. Does $S\bigcup T$ forms a subspaces of $\mathbb{R}^3$?

(b) Examine if the set S is a subspace of $\mathbb{R}^3$, where $S=\{(x,y,z)\in\mathbb{R}^3: xy=z\}$.

18. Show that the set $\{1,x,x^2,.....,x^n,.....\}$ is a basis of the vector space all polynomials $P(\mathbb{R})$. (5)

19. (a) Let V and W be vector spaces over a field F . Let $T:V\to W$ be a linear transformation . Show that T is one-to-one if and only if $\ker T=\{\theta\}$, where θ is the additive identity of V . (3)

(b) Check whether $T:\mathbb{R}^2\to\mathbb{R}^2$ defined by $T(a_1,a_2)=(a_1+1, 2a_2)$ is a linear transformation or not. (2)

20. (a) Let $T:P_2(\mathbb{R})\to P_3(\mathbb{R})$ be a linear transformation defined by $T(f(x))=2f'(x)+\int_0^x 3f(t)\,dt$.

Does T is one-to-one and onto ?

(b) State the Rank-Nullity theorem .

21. (a) Suppose that $T:\mathbb{R}^2\to\mathbb{R}^2$ is linear , $T(1,0)=(1,4)$ and $T(1,1)=(2,5)$ .What is $T(2,3)$? (2)

(b) Let , $T:M_{2\times 2}(\mathbb{R})\to P_2(\mathbb{R})$ be a linear transformation defined by $T\begin{pmatrix} a & b \\ c & d\end{pmatrix}=(a+b)+2dx+bx^2$.

Let $\beta=\left\{\begin{pmatrix}1&0\\0&0\end{pmatrix},\begin{pmatrix}0&1\\0&0\end{pmatrix},\begin{pmatrix}0&0\\1&0\end{pmatrix},\begin{pmatrix}0&0\\0&1\end{pmatrix}\right\}$ and $\gamma=\{1,x,x^2\}$ be the standard basis of $M_{2\times 2}(\mathbb{R})$ and $P_2(\mathbb{R})$ respectively . Compute $[T]_\beta^\gamma$. (3)

22. Extend the set S to obtain a basis of the vector space of $\mathbb{R}^3$, where $S = \{(1,2,1),(2,1,1)\}$. (5)

23. Show that $\mathbb{R}$ is a vector space over $\mathbb{Q}$, where $\mathbb{Q}$ denotes the set of all rational numbers. Is $\mathbb{Q}$ is a vector space over $\mathbb{R}$? Justify. (3+2)

24. Determine the linear mapping $T:\mathbb{R}^3 \to \mathbb{R}^3$ that maps the basis vectors (0,1,1) , (1,0,1) , (1,1,0) of $\mathbb{R}^3$ to the vectors (2,1,1) , (1,2,1) , (1,1,2) respectively . Verify that dim KerT + dim ImT = 3 . (3+2)

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